

THE REACTION $\pi N \rightarrow \pi\pi N$ At THRESHOLD

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I summarize the results of the complete one-loop chiral perturbation theory calculation performed recently. It is shown that it allows to accurately pin down the isospin two, S-wave $\pi\pi$ scattering length a_0^2 . On the other hand, interesting resonance physics makes a precise determination of a_0^0 very difficult.

1 PRELIMINARIES

The interest in the reaction $\pi^a(k) + N(p_1) \rightarrow \pi^b(q_1) + \pi^c(q_2) + N(p_2)$, where ' a, b, c ' are pion isospin indices and N denotes the nucleon (neutron or proton), stems mostly from the fact that it includes (besides many other contributions) the one-pion exchange diagram with the four-pion vertex. This allows to extract information about the on-shell low-energy $\pi\pi$ interaction which is the purest testing ground of our understanding of the spontaneous and explicit chiral symmetry breaking in QCD. The relation between the threshold $\pi\pi N$ and the threshold $\pi\pi$ amplitudes has until recently only been known at lowest order (tree level). To precisely determine the $\pi\pi$ S-wave scattering lengths a_0^I (with $I = 0$ or 2), a one-loop calculation within the framework of heavy nucleon chiral perturbation theory is called for. I will present the most salient results of a recent investigation of that type here (for details, see Ref.[1]).

At threshold, the on-shell amplitude in the $\pi^a N$ cms can be expressed in terms of two threshold amplitudes, called D_1 and D_2 ,

$$T = i\vec{\sigma} \cdot \vec{k} \left[D_1 (\tau^b \delta^{ac} + \tau^c \delta^{ab}) + D_2 \tau^a \delta^{bc} \right], \quad (1)$$

where $\vec{\sigma}$ denotes the spin vector of the initial nucleon. The chiral expansion of the amplitudes $D_{1,2}$ takes the form

$$D_{1,2} = (f_0)_{1,2} + (f_1)_{1,2} \mu + (f_2)_{1,2} \mu^2 + \dots, \quad \mu \equiv M_\pi/m, \quad (2)$$

modulo logs and M_π (m) denotes the pion (nucleon) mass.

2 LOW-ENERGY THEOREMS FOR $\pi N \rightarrow \pi\pi N$

In Ref.[2], the first two terms of the chiral expansion, Eq.(2), were calculated. This amounted to a set of novel low-energy theorems (LETs) since in the resulting expressions only well-known parameters appear,

$$D_1 = \mathcal{C} \left[1 + \frac{7}{2} \mu \right] = 2.4 \text{ fm}^3, \quad D_2 = -\mathcal{C} \left[3 + \frac{17}{2} \mu \right] = -6.8 \text{ fm}^3, \quad (3)$$

with $\mathcal{C} = g_{\pi N}/(8mF_\pi^2)$. In Refs.[2,3], the threshold data (i.e. for pion kinetic energies less than 30 MeV above threshold) for the two channels $\pi^+p \rightarrow \pi^+\pi^+n$ (which is sensitive to D_1) and $\pi^-p \rightarrow \pi^0\pi^0n$ (which is sensitive to D_2) were fitted,

$$D_1^{\text{exp}} = 2.26 \pm 0.12 \text{ fm}^3, \quad D_2^{\text{exp}} = -9.05 \pm 0.36 \text{ fm}^3, \quad (4)$$

which shows the expected pattern of deviation from the LETs, namely small/sizeable for the isospin two/zero $\pi\pi$ final state (since D_1 is directly proportional to the $I_{\pi\pi} = 2$ amplitude). Notice that more global fits including data up to much higher energies tend to give somewhat different values for $D_{1,2}^{\text{exp}}$ because the threshold region does not have sufficient statistical weight in such global fits (see e.g. Ref.[4]). At this order, however, the $\pi\pi$ interaction to be extracted does not come into play, i.e. one has to go one order higher and calculate *all* terms of order M_π^2 .

3 OUTLINE OF THE CALCULATION TO ORDER M_π^2

Here, I can just give a flavor of the rather involved calculation to $\mathcal{O}(M_\pi^2)$ presented in Ref.[1]. There are essentially four types of contributions which have to be considered. These are related to the effective pion–nucleon Lagrangian $\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)}$ as follows (where the superscript ' i ' gives the chiral dimension):

- *One-loop graphs with insertions from $\mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{\pi\pi}^{(2)}$* : Including mass and coupling constant renormalization, there are 36×2 topologically different one loop graphs to be considered (the factor two represents the interchange of the two final-state pions). Some of the rescattering diagrams lead to an imaginary part at threshold since they are evaluated at $\omega = 2M_\pi$ well above the branch cut at $\omega_0 = M_\pi$.
- *Tree graphs with insertions from $\mathcal{L}_{\pi\pi}^{(4)}$* : These are the ones which are sensitive to the pion–pion interaction at next-to-leading order which was studied in great detail by Gasser and Leutwyler [5].
- *Tree graphs with insertions from $\mathcal{L}_{\pi N}^{(2)}$* : These appear only with an extra suppression factor of $1/m$ (otherwise they would have already shown up in the LETs, Eq.(3)). The corresponding low-energy constants can be fixed by calculating the subthreshold expansion of the elastic πN scattering amplitudes and comparing these to the empirical values as given e.g. by Höhler [6].
- *Tree graphs with insertions from $\mathcal{L}_{\pi N}^{(3)}$* : These are in fact the ones which are the hardest to pin down. Since there are not enough data to fix them all, they are estimated in Ref.[1] by resonance exchange (baryon excitations in the s -channel and meson excitations in the t -channel). It turns out that for the threshold amplitudes, only the excitation of the Roper $N^*(1440)$ is of relevance, see also Ref.[7].

Before presenting results, let me elaborate a bit on the Roper contribution. While the $N^*N\pi$ vertex is fairly well known, it was shown in Ref.[1] that for the $N^*N(\pi\pi)_S$ vertex there are effectively two *different* lowest order couplings (written here non-relativistically),

$$\mathcal{L}_{N^*N\pi\pi} = c_1 M_\pi^2 N^* \vec{\pi}^2 N + c_2 N^* (v_\mu \partial^\mu \vec{\pi})^2 N \quad (5)$$

where v_μ is the four-velocity of the heavy baryon. Both terms in Eq.(5) have chiral dimension two. Since only the branching ratio $N^* \rightarrow N(\pi\pi)_S$ is known, performing the phase space integration leads to a quadratic form in the coupling constants c_1 and c_2 , defining an ellipse. This ellipse is rather elongated and thus leads to a large uncertainty in the sum $c_1 + c_2$ since the energy-dependence of the second coupling in Eq.(5) gives rise to a substantial enhancement compared to the energy-independent coupling. It remains a challenge to disentangle these coupling constants and further pin down their numerical values. Furthermore, the N^* contributes solely to D_2 .

3 RESULTS FOR THE $\pi N \rightarrow \pi\pi N$ THRESHOLD AMPLITUDES

Consider first the amplitude D_1 . Adding all theoretical uncertainties in quadrature (for details, see Ref.[1]), one finds

$$D_1 = 2.65 \pm 0.24 \text{ fm}^3, \quad (6)$$

which overlaps within one standard deviation with the empirical value, Eq.(4). It is worth to note that the theoretical uncertainty is larger than the empirical one. Most interesting, however, is the chiral expansion of D_1 , i.e. its separation into the contributions of order one, M_π and M_π^2 (modulo logs),

$$D_1 = 1.59 \cdot (1.00 + 0.52 + 0.15) \text{ fm}^3, \quad (7)$$

which shows a rapid convergence. The first three terms in the chiral expansion behave like $1.59 \exp(0.52)$, so one expects the correction at order M_π^3 to be very small. Note, however, that such an estimate can not substitute for a full calculation to higher order.

In the case of D_2 , matters are different. The central value including all corrections to $\mathcal{O}(M_\pi^2)$ comes out surprisingly close to the empirical value (cf. Eq.(4)),

$$D_2 = -9.06 \pm 1.05 \text{ fm}^3, \quad (8)$$

where the large uncertainty is mostly due to the Roper as discussed before. While one might content oneself with such a result, a closer look at the chiral expansion

$$D_2 = -4.76 \cdot (1.00 + 0.42 + 0.48) \text{ fm}^3, \quad (9)$$

reveals that there are still large corrections at $\mathcal{O}(M_\pi^2)$. So only a calculation to (at least) one more order in the chiral expansion (which is still one loop) can clarify whether the nice agreement between theory and experiment for D_2 is accidental or not. Such a calculation is not yet available and can only be performed if one clarifies before the nature and strengths of certain resonance couplings entering at that order (like the Roper already discussed but also the $N\Delta\pi\pi$ coupling and so on).

4 RESULTS FOR THE S-WAVE $\pi\pi$ SCATTERING LENGTHS

While the isospin two S-wave $\pi\pi$ scattering length is directly proportional to D_1 , the value for a_0^0 has to be extracted from the combination $-2D_1 - 3D_2$. We can therefore expect to determine a_0^2 rather precisely whereas the resulting value for a_0^0 has to be considered indicative due to the possible large higher order corrections in D_2 . Adding the theoretical and empirical uncertainties in quadrature, one finds

$$a_0^0 = 0.21 \pm 0.07, \quad a_0^2 = -0.031 \pm 0.007, \quad (10)$$

which both are in good agreement with the one-loop chiral perturbation theory prediction of Gasser and Leutwyler [5],

$$a_0^{0,\text{CHPT}} = 0.20 \pm 0.01, \quad a_0^{2,\text{CHPT}} = -0.042 \pm 0.008. \quad (11)$$

Furthermore, the value for the combination $2a_0^0 - 5a_0^2 = 0.577 \pm 0.144$ is consistent with the so-called universal curve (UC), $(2a_0^0 - 5a_0^2)_{\text{UC}} = 0.614 \pm 0.028$. The numbers given here supersede the previously determined $\pi\pi$ scattering lengths from the threshold $\pi\pi N$ amplitudes based on the Olsson–Turner model [8] which is only compatible with QCD for $\xi = 0$ (since by now we know that the symmetry breaking is $\bar{3} \times 3$) and thus amounts to the tree level approach.

5 SHORT SUMMARY

The reaction $\pi N \rightarrow \pi\pi N$ at threshold has been evaluated within the framework of chiral perturbation theory to one loop accuracy [1]. It appears to be well suited for a precise determination of the isospin two S-wave pion–pion scattering length a_0^2 . In case of the isospin zero scattering length a_0^0 , there are still large theoretical uncertainties related to interesting baryon resonance physics which make an accurate determination very difficult. However, it is worth noticing the stunning agreement of the extracted value for a_0^0 with the one-loop CHPT prediction [5] – is that merely an accident? More theoretical effort is needed to clarify this issue. At present, it appears that $K_{\ell 4}$ -decays or pionic molecules are better candidates to precisely pin down a_0^0 .

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References

1. V. Bernard, N. Kaiser, and Ulf-G. Meißner, *Nucl. Phys. B* (1995) in print.
2. V. Bernard, N. Kaiser and Ulf-G. Meißner, *Phys. Lett. B* **332**, 415 (1994);
ibid (E) **B338**, 520 (1994).
3. V. Bernard, N. Kaiser, and Ulf-G. Meißner, *Int. J. Mod. Phys. E* **4**, 193 (1995).
4. H. Burkhard and J. Lowe, *Phys. Rev. Lett.* **67**, 2622 (1991).
5. J. Gasser and H. Leutwyler, *Ann. Phys. (NY)* **158**, 142 (1984).
6. G. Höhler, in Landolt-Börnstein, vol.9b2, H. Schopper (ed.), Springer, Berlin (1983).
7. D.M. Manley et al., *Phys. Rev. D* **30**, 904 (1984).
8. M.G. Olsson and Leaf Turner, *Phys. Rev. Lett.* **20**, 1127 (1968).